
Jet and Rocket Propulsion

AE4451

LECTURE 11

Overview

what we saw last time:

- cycle analysis
 - in-class example determining product composition where iteration necessary
 - idea:
 - find mole fractions using imposed combustion temperature
 - find enthalpy release and compare to expected
 - if different, give another temperature estimate and recalculate

today:

- efficiencies and other performance metrics
- performance progress for air-breathing technologies

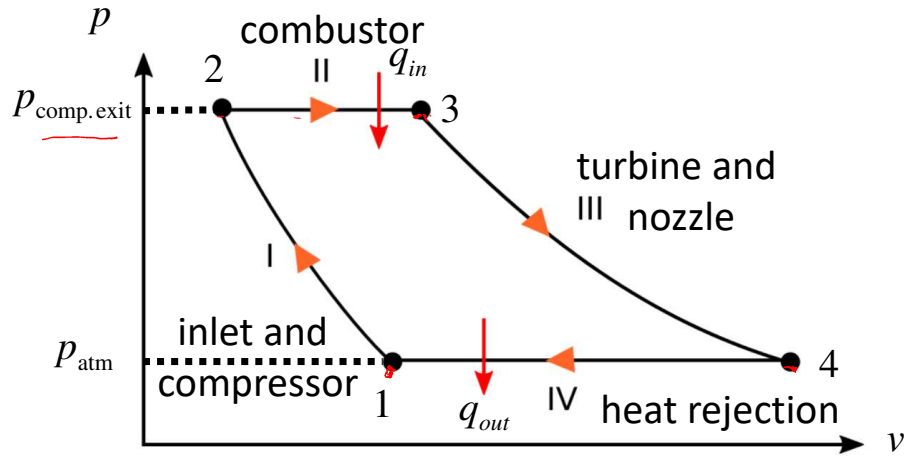
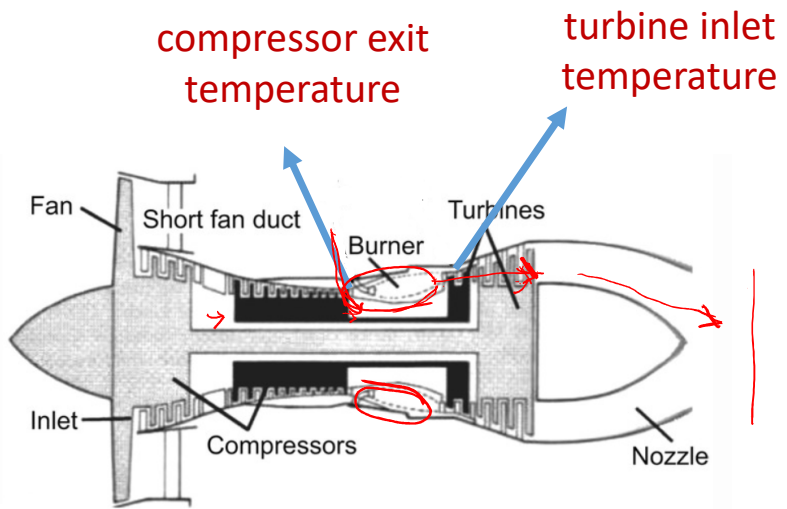
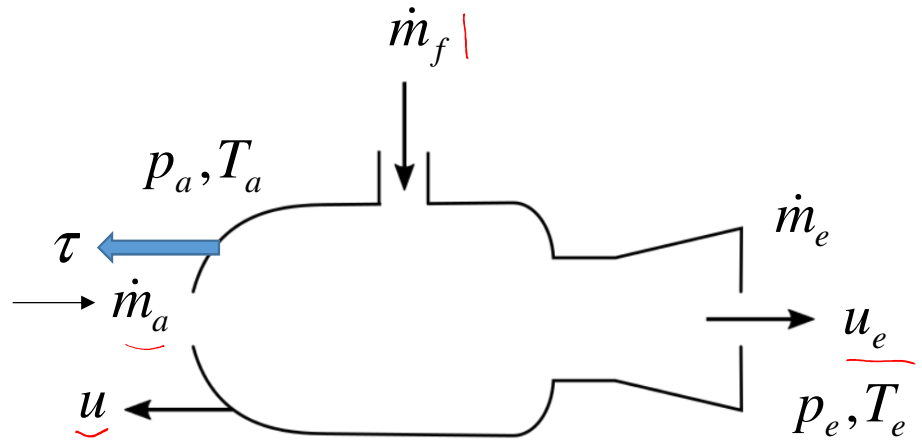
Air-breathing system performance

- Thermal efficiency

efficiency of conversion of fuel energy to k.e or shaft work

$$\eta_{th} = \frac{(1+f)u_e^2 - u^2}{2f\Delta h_R} \quad \eta_{th} \equiv \frac{\dot{W}_{shaft}}{\dot{m}_f \Delta h_R}$$

fuel to air ratio (f)

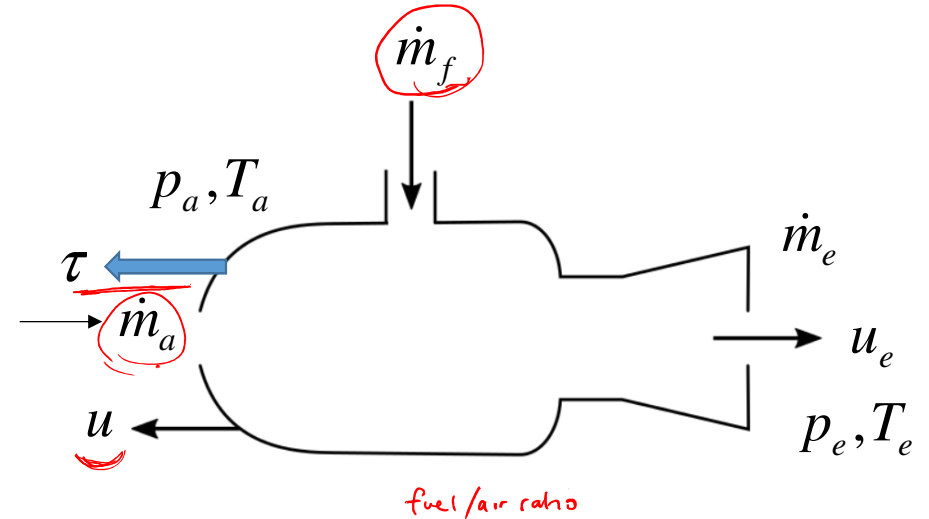


Air-breathing system performance

- Propulsive efficiency

a measure of how effective the system is in producing thrust

$$\eta_p = \frac{\tau u}{\Delta \dot{KE}}$$



$$\Delta \dot{KE} = \frac{1}{2} (\dot{m}_a + \dot{m}_f) u_e^2 - \frac{1}{2} \dot{m}_a u^2$$

while specific thrust is

$$\frac{\tau}{\dot{m}_a} = [(1+f)u_e - u] + \frac{(p_e - p_a)A_e}{\dot{m}_a}$$

if $p_e = p_a$ then
$$\eta_p = \frac{2(\tau/\dot{m}_a)u}{(1+f)u_e^2 - u^2}$$

or
$$\eta_p = 2 \frac{(1+f)u_e/u - 1}{(1+f)(u_e/u)^2 - 1}$$

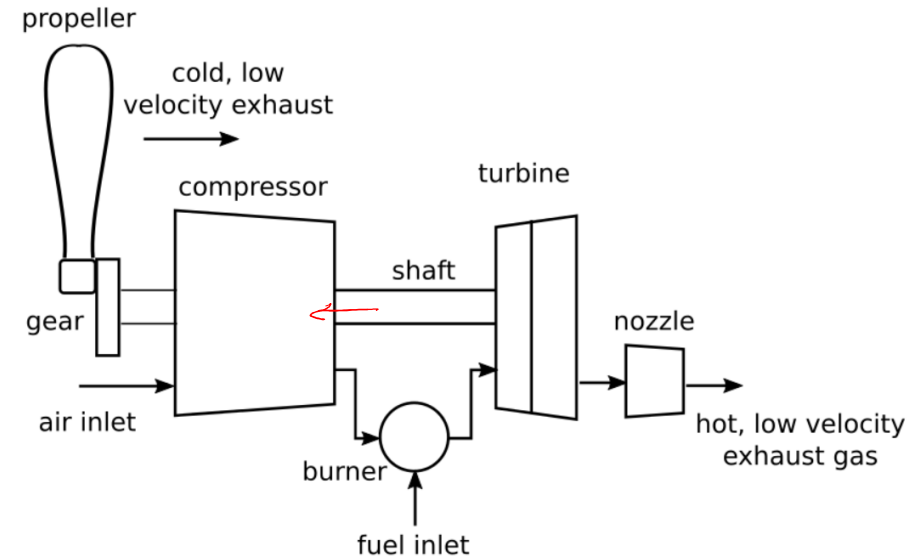
Air-breathing system performance

- Propulsive efficiency

- for turboprop engines, it is typical to replace propulsive efficiency with a **propeller efficiency**

$$\eta_{pr} \equiv \frac{\tau_{pr} u}{\dot{W}_{shaft}}$$

τ_{pr}
thrust from propeller



- if turboprop derives significant thrust from an engine exhaust nozzle (in addition to the propeller), sometimes useful to define an equivalent shaft power

$$\dot{W}_{shaft, equiv} = \dot{W}_{shaft} \left(1 + \frac{\tau_{nozzle}}{\tau_{pr}} \right)$$

$$\text{then } \eta_{pr} \equiv \frac{\tau u}{\dot{W}_{shaft, equiv}}$$

Air-breathing system performance

• Propulsive efficiency η_p

$u_e \sim u$
 $\eta_p = \frac{2(1+f)u_e/u - 1}{(1+f)(u_e/u)^2 - 1}$

$$\eta_p = 2 \frac{(1+f)u_e/u - 1}{(1+f)(u_e/u)^2 - 1}$$

- f of 0.01 and 0.5
- here, significant increase in f leads to only minimal change to efficiency
- efficiency decreasing as $u_e \gg u$

not eat m_a

f negligible

$$2 \frac{(u_e/u - 1)}{(u_e/u)^2 - 1}$$

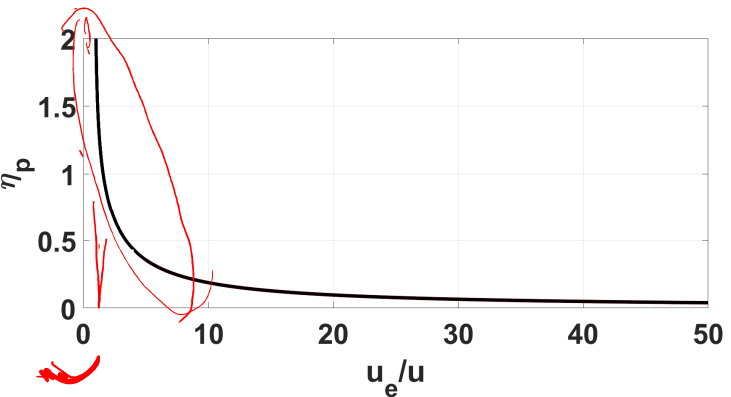
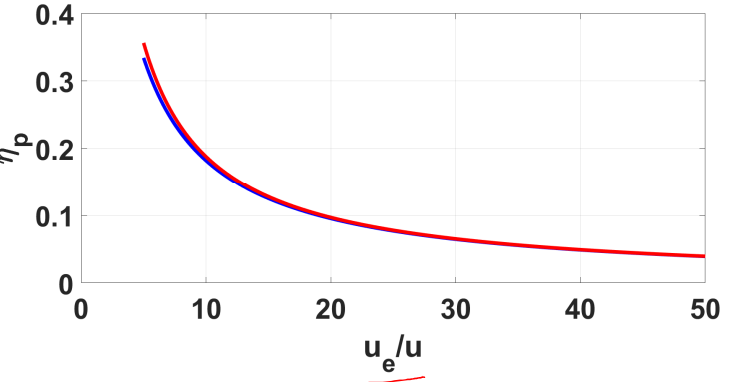
$$2 \frac{(u_e/u - 1)}{(u_e/u - 1)(u_e/u + 1)}$$

- f of 0.5
- here, as $u_e \rightarrow u$, efficiency increasing
- η_p can be > 1 since fuel is being ejected

$$A_p = \frac{2}{\left(\frac{u_e}{u} + 1\right)}$$

low ratio u_e/u

$$\frac{2 + 2f - 1}{f} = \frac{1}{f} + 2$$



$u_e \sim u$
 large ratio

Air-breathing system performance

- Simplified expressions for the propulsive and thermal efficiencies

$$\eta_p = 2 \frac{(1+f)u_e/u - 1}{(1+f)(u_e/u)^2 - 1} \quad \text{assuming } f \ll 1 \text{ (usually the case),}$$

*fuel
air ~ few %*

$$\eta_p \approx 2 \frac{u_e/u - 1}{(u_e/u)^2 - 1} = 2 \frac{(u_e/u - 1)}{(u_e/u + 1)(u_e/u - 1)} \quad \text{thus } \boxed{\eta_p \approx \frac{2u}{u_e + u}}$$

This assumption also gives, for the thermal efficiency,

$$\eta_{th} = \frac{(1+f)u_e^2 - u^2}{2f\Delta h_R} \Rightarrow \boxed{\eta_{th} = \frac{u_e^2 - u^2}{2f\Delta h_R}} \quad f \ll 1$$

Air-breathing system performance

- Let's consider both efficiency contributions

in terms of the kinetic energy, we saw earlier:

$$\eta_{th} = \frac{\Delta \dot{KE}}{m_f \Delta h_R} \quad \text{and} \quad \eta_p = \frac{\tau u}{\Delta \dot{KE}}$$

previously, we saw the overall efficiency

$$\eta_o = \frac{\tau u}{\dot{m}_f \Delta h_R}$$

i.e. thrust power/heating rate from fuel
for thrust producing engines

or, simply:

$$\eta_o = \eta_{th} \eta_p$$

(for turboprop)

$$\eta_o = \eta_{th} \eta_{pr}$$

$$(\eta_o = \eta_{th} \eta_p \eta_{tr})$$

η_{tr} transmission efficiency close to 1

i.e., we only need to know two of the efficiencies to find the third

Specific fuel consumption, SFC

- How much does a given amount of thrust cost in fuel?

lower specific fuel consumption (SFC)



greater aircraft range

thrust specific fuel consumption (TSFC),
or just SFC

$$TSFC \equiv \frac{\dot{m}_f}{\tau}$$

since $\eta_o \equiv \frac{\tau u}{\dot{m}_f \Delta h_R}$ *overall efficiency*

$$TSFC = \frac{u}{\eta_o \Delta h_R}$$

can also write TSFC in terms of the specific thrust,

$$TSFC = \frac{\dot{m}_f / \dot{m}_a}{\tau / \dot{m}_a} = \frac{f}{\underline{\underline{ST}}} \quad \text{fuel/air ratio}$$

Specific fuel consumption, SFC

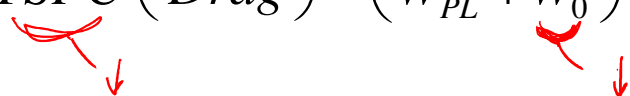
- as before, we need to consider equivalent expressions for turboshaft engines,

$$BSFC \equiv \frac{\dot{m}_f}{\dot{W}_{shaft}} \quad \text{"brake specific fuel consumption (BSFC)"}$$

- this metric can be applied to any type of fuel-burning (combustion) engine that produces shaft power (diesels, spark-ignition, ...)
- “brake” is a hold-over name from the way that shaft-power was typically tested on a dynamometer where the shaft power is absorbed (a “brake”) and measured

Aircraft range

- the aircraft range is another way to express the aircraft fuel efficiency
- it can be calculated from the **Breguet range equation**:

$$\text{aircraft range} = \frac{u}{TSFC} \left(\frac{Lift}{Drag} \right) \ln \left(\frac{W_{fuel}}{W_{PL} + W_0} \right)$$


W_{fuel} = fuel weight

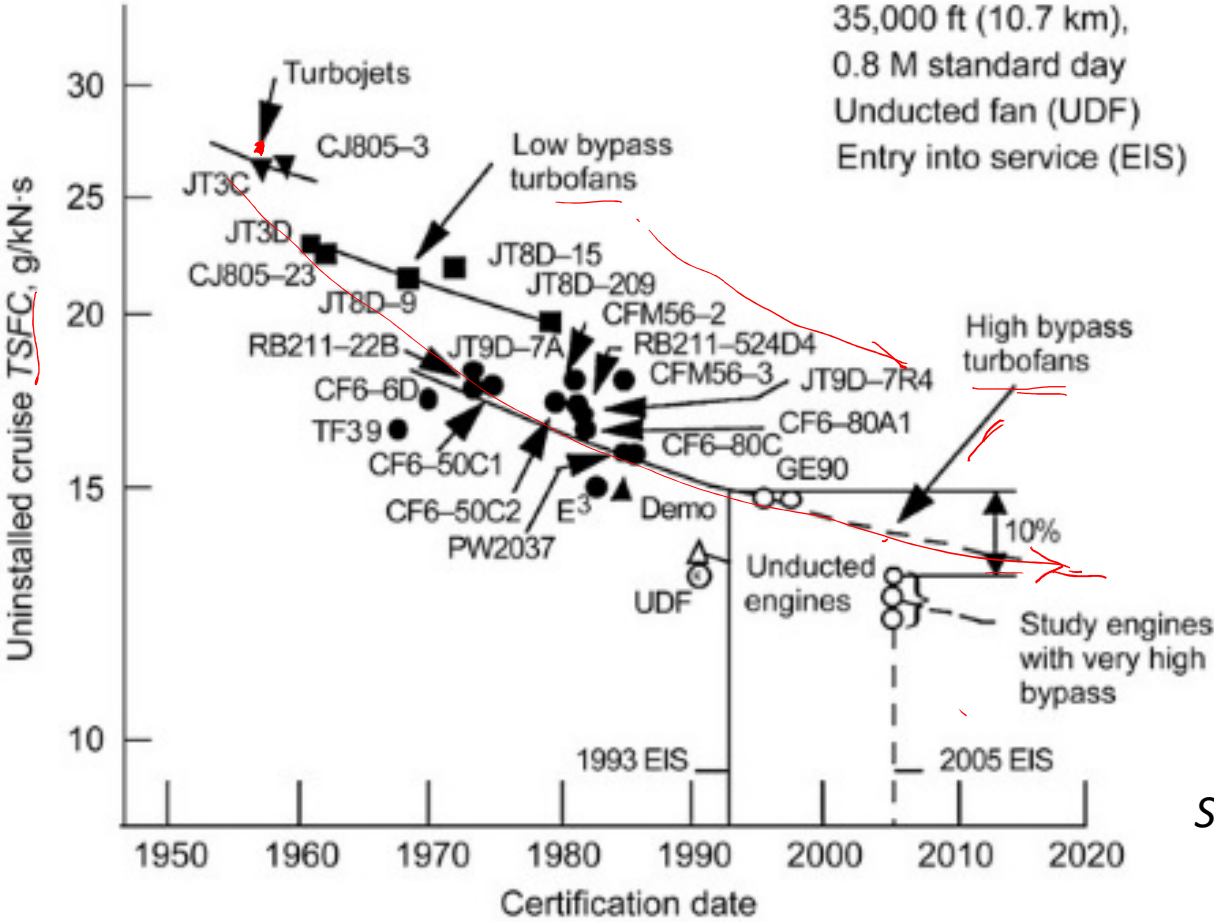
W_{PL} = payload weight

W_0 = aircraft empty weight

- improvements in range chiefly from:
 - decrease in TSFC
 - decrease in engine weight

Jet engine performance history

- uninstalled TSFC versus engine certification date



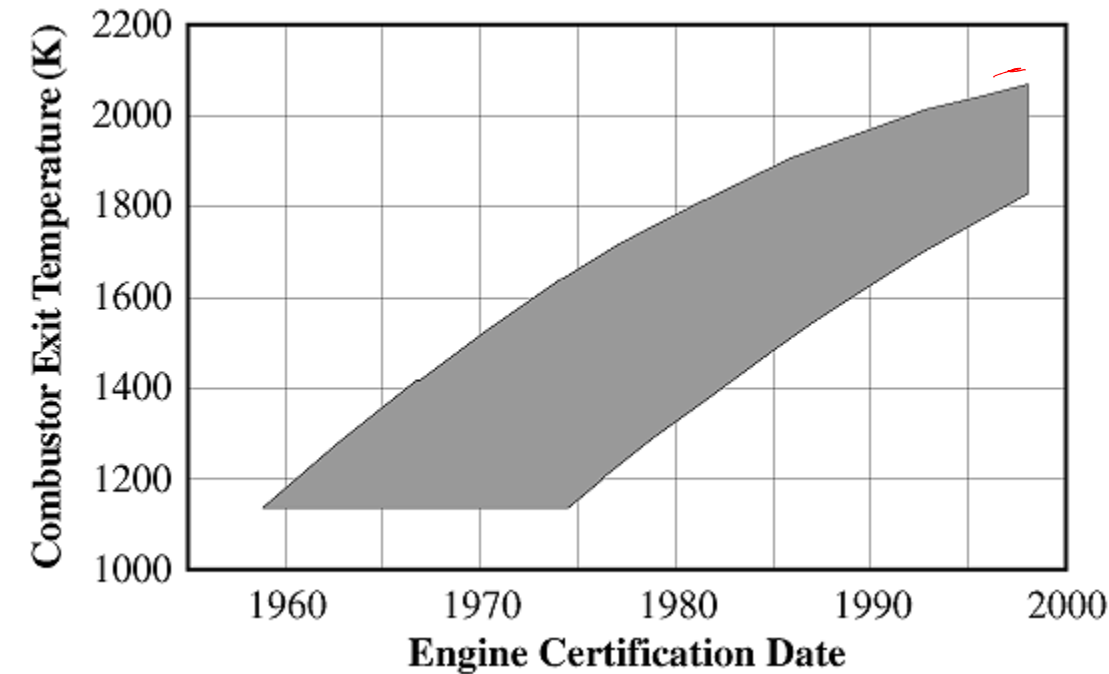
$$TSFC \equiv \frac{\dot{m}_f}{\tau}$$

"uninstalled" thrust: measured with thruster system not attached to vehicle

Suder and Heidmann, 2018

Jet engine performance history

- combustor exit temperature



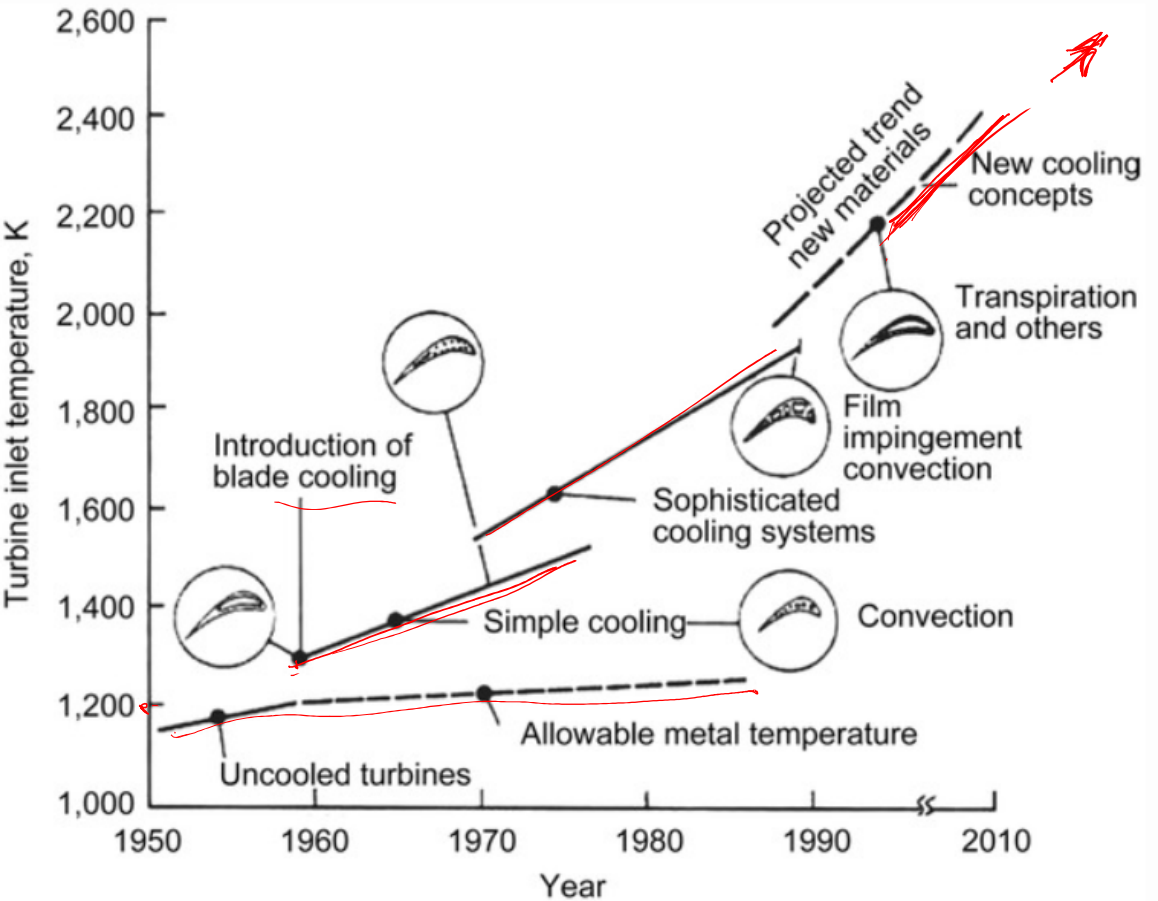
$$\eta_B = 1 - \frac{T_1}{T_2} = 1 - \frac{T_{\text{atmosphere}}}{T_{\text{compressor exit}}}$$

Handwritten notes:
 - η_B is labeled as η_{th} (thermal efficiency) and η_B (cycle efficiency).
 - $T_{\text{compressor exit}}$ is circled in red.
 - A red arrow points down from the circled term.

- notice this **thermal** efficiency was based on the compressor exit temperature (< the combustor exit temperature)
- compressor exit temperature limited to ~ 1000 K
- turbine inlet temperature limited to ~ 2000 K

Jet engine performance history

- turbine inlet temperature



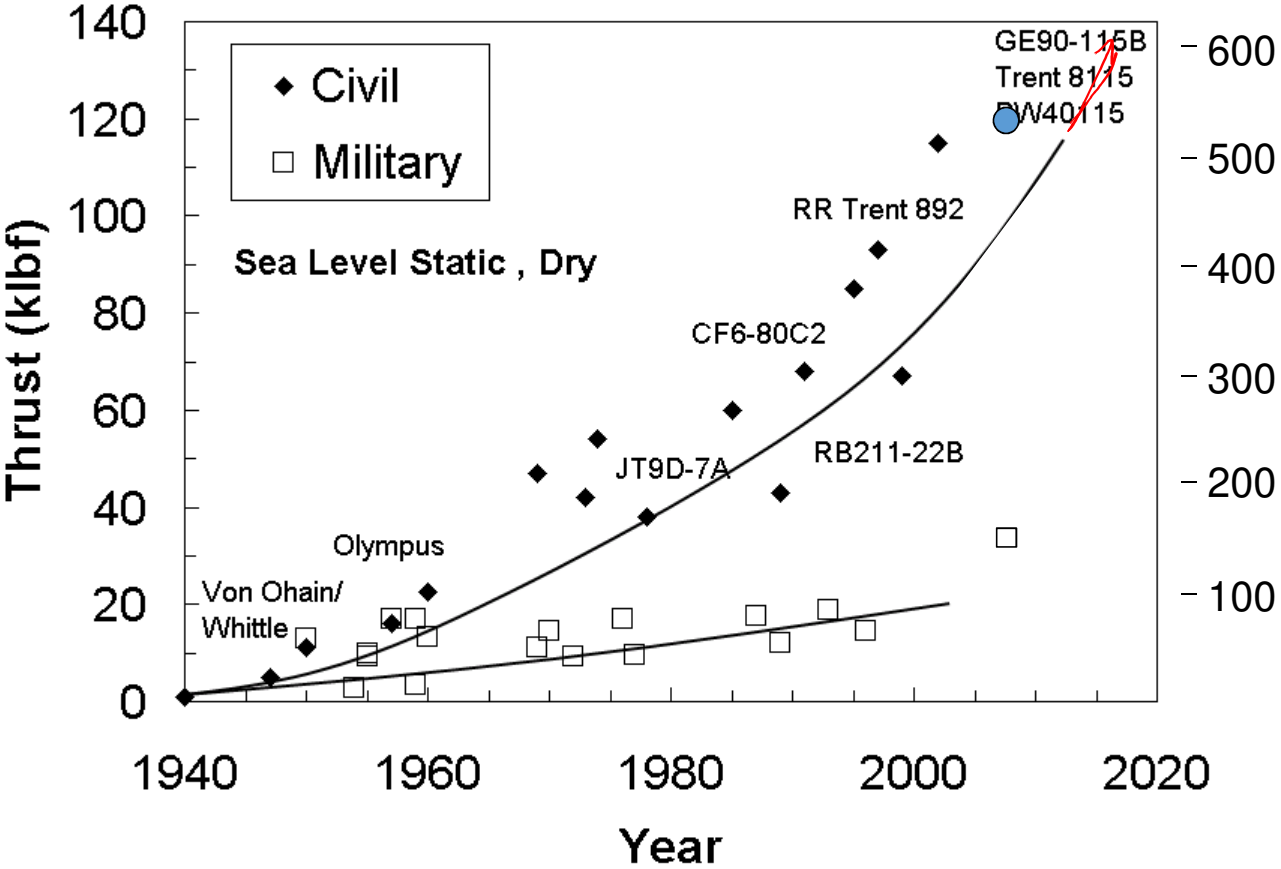
- advanced cooling systems have allowed the turbine inlet temperatures to be increased

recently: thermal barrier coatings (ceramic coatings providing insulation)

- in future: ceramic base materials

Jet engine performance history

- evolution in thrust level

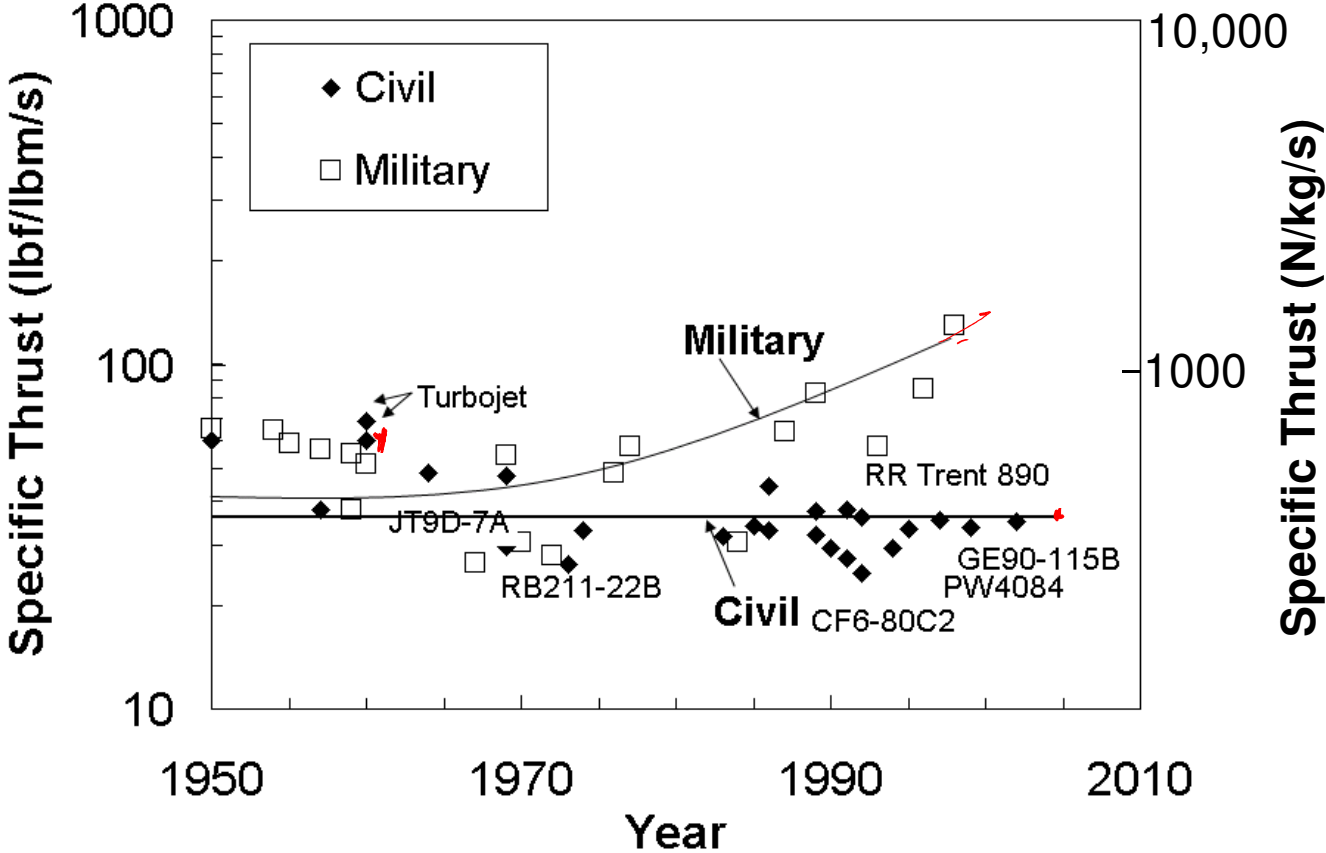


Ballal and Zelina, 2003

- since 1939, static thrust increase by $> 110 \times$ for civil engines and $20 \times$ for military engines
- 1903:
 - 134 lbf Wright Flyer
- 1939:
 - 1,000 lbf (~4450N) von Ohain/Whittle
- 2004:
 - 35,000 lbf Military Engine
 - 115,000 lbf (GE90-115B)
 - tested up to 120,000 lbf

Jet engine performance history

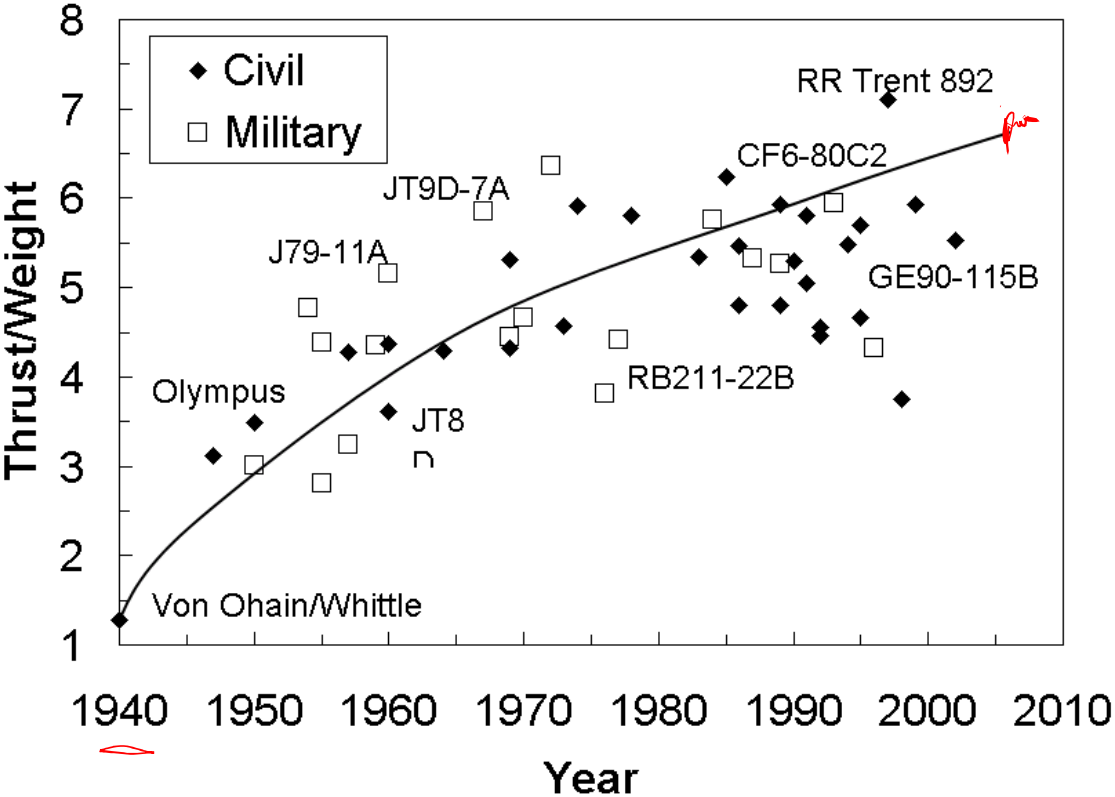
- evolution in specific thrust level



Ballal and Zelina, 2003

Jet engine performance history

- thrust-to-weight evolution



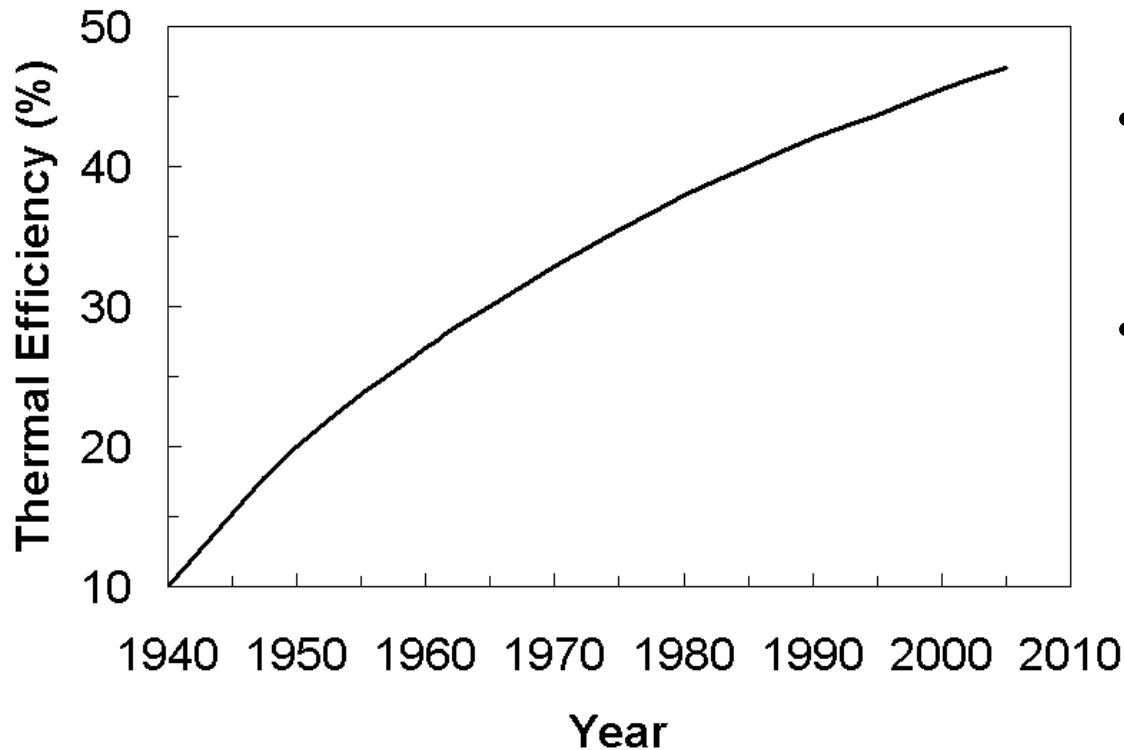
- 1903: 0.67
Wright Flyer
- 1939: 1.2
von Ohain/Whittle
- 2003: 6.5
Military Engine
- 2003: 7
Rolls Royce Trent

for comparison, ~85 for Saturn V F-1 Engine

Ballal and Zelina, 2003

Jet engine performance history

- evolution in efficiency



$$\eta_{th} = \frac{\dot{\Delta KE}}{m_f \Delta h_R}$$

- modern aeroengine thermal efficiency approaching 50%

- for overall efficiency $\eta_o \equiv \frac{tu}{\dot{m}_f \Delta h_R}$

1903: 10% (Wright Flyer)

1939: 15% (von Ohain and Whittle)

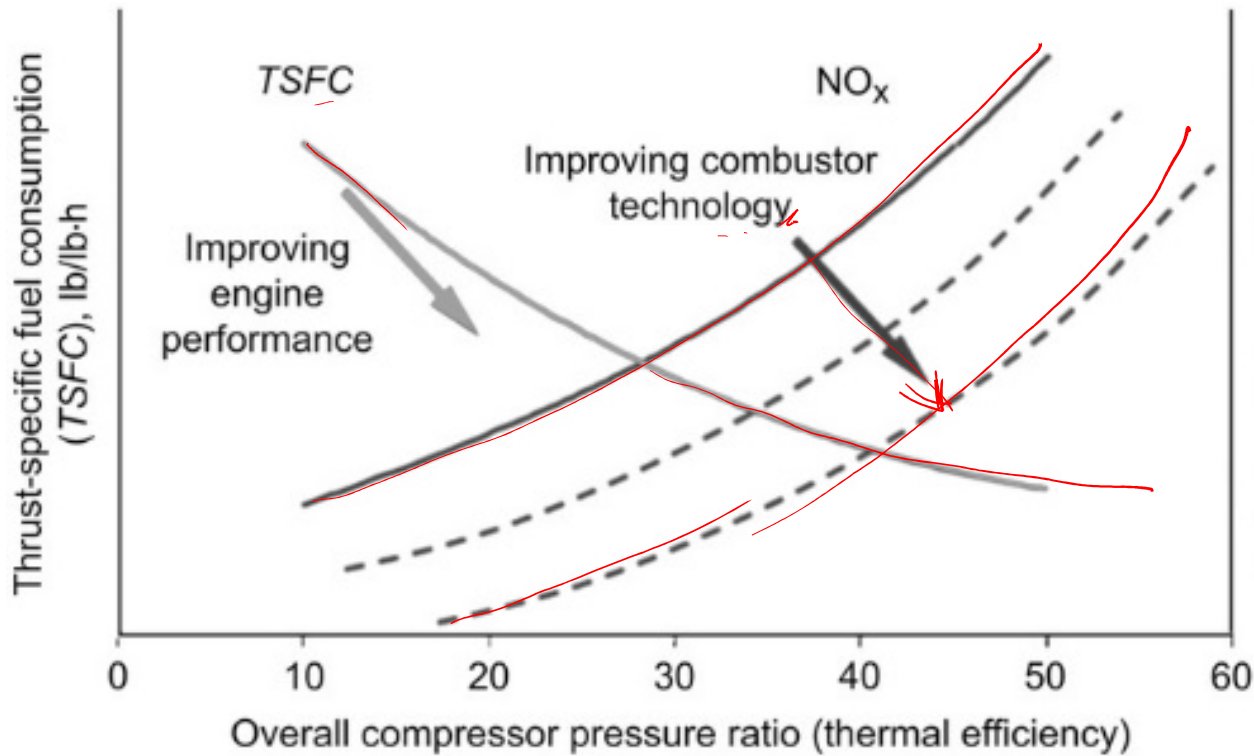
2003: 30% (Military Engine)

40% (Civil Engine)

Ballal and Zelina, 2003

Jet engine performance history

- evolution in emissions



- increasing cycle temperature good for thermal efficiency, but increases NOx emissions
- NOx from nitrogen and oxygen reaction during combustion of fuels in air

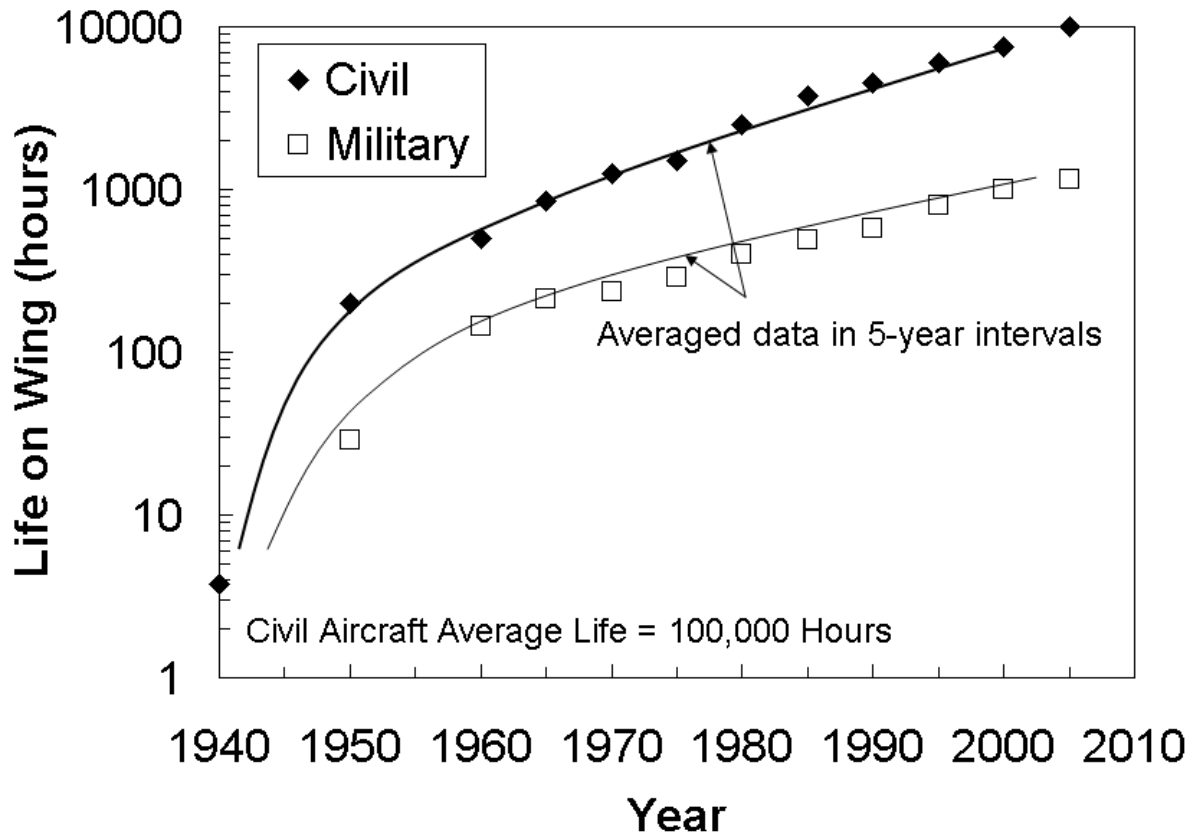
$$* \eta_B = 1 - \frac{T_1}{T_2} = 1 - \frac{T_{\text{atmosphere}}}{T_{\text{compressor exit}}}$$

$$\eta_B = 1 - \frac{1}{TR} = 1 - \frac{1}{PR^{(\gamma-1)/\gamma}}$$

Routledge, 2017

Jet engine performance history

- life on wing

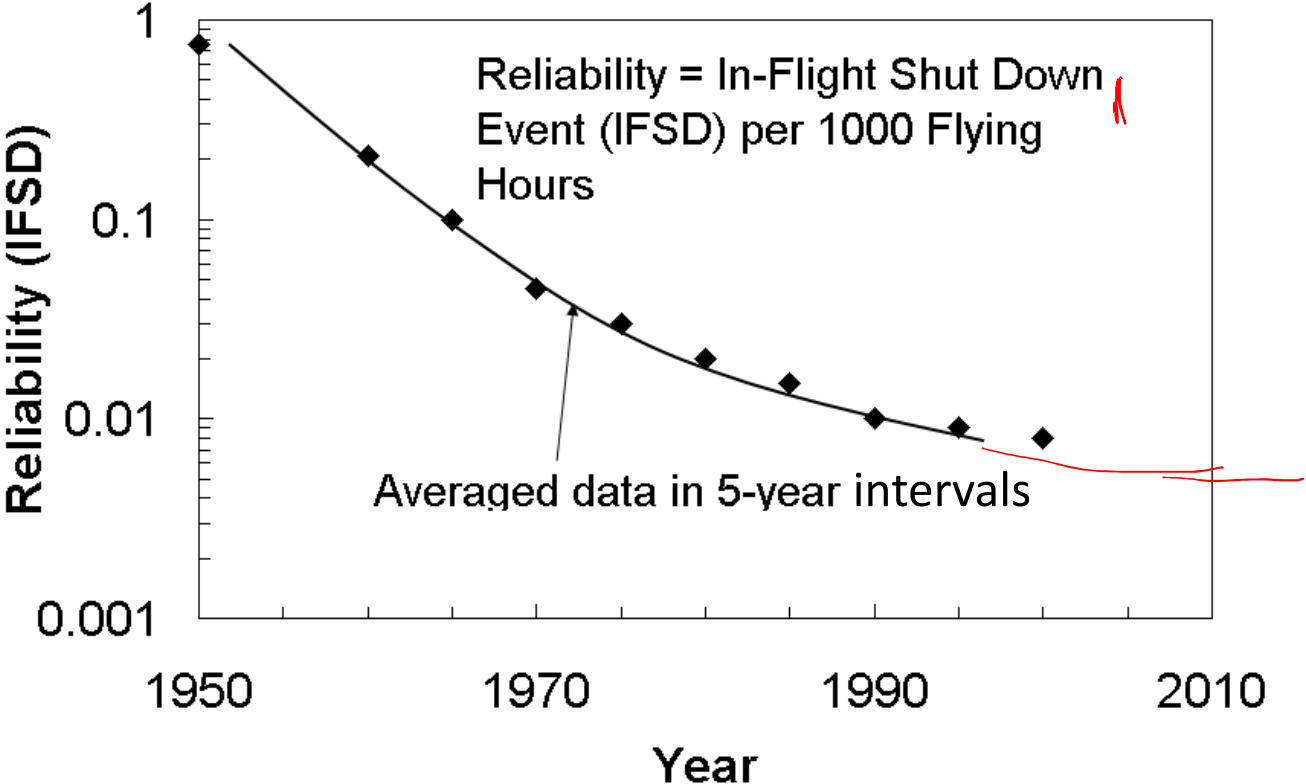


- early jet engines lasted < 10 hours
- modern civil engines can stay on wing for > 10,000 hours
- military engines last up to 800 hours

Ballal and Zelina, 2003

Jet engine performance history

- reliability



Ballal and Zelina, 2003